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Pull Up a Chair: Municipal Council Size and Local Taxes in Brazil*

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Abstract

Does legislative size affect the diversity of political representation and a government's attitude toward taxation? To answer this, we exploit a Brazilian reform that allocated the number of seats on municipal councils based on population thresholds. Using a regression discontinuity design, we find that councils with an additional member have greater political diversity, consistent with Duverger's Law. Larger councils have significantly higher tax revenues, with this increase being driven by a less salient form of tax - on services - as compared to property taxes. More diverse councils use the increased tax revenue on "pro-social" public goods: health, housing, and social assistance.

Keywords: Municipal Councils; Local Taxation; Council Size; Regression Discontinuity

JEL Classification Numbers: D72, H71

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The ability of local governments to raise tax revenue is a central issue to the building of state capacity and the necessary mobilization of resources for the provision of public goods and services to a broad share of the population. The level of efficiency under which different governments perform such tasks is uneven across countries (Besley and Persson, 2014), and depends on the size and growth of local governments (Miller and Moe, 1983). On the one hand, larger local governments might create a weak "local Leviathan" - an overextended and poor bureaucratic system - with little checks on its growth and inefficient use of the available resources. On the other hand, a larger number of representatives might increase diversity in political participation, with a higher share of the population being enfranchised and represented. Political diversity, by its turn, might ensure that the increased plurality of views and needs are effectively addressed, creating the necessary political incentives and checks on the elected representatives to efficiently deliver the public goods and services to their electorate.

This paper examines whether local legislature size affects political representation, and subsequently a governments' ability to raise tax revenue. We take advantage of a 2004 Brazilian reform that allocated the number of seats on a municipal council based on population thresholds. Employing a Regression Discontinuity (RD) design, we find that an additional seat on a municipal council improves political diversity, with an average increase of one additional "effective number of parties" holding a seat.

Additionally, municipalities with an extra seat collect significantly higher tax revenues than municipalities on the lower side of the threshold. The increase in tax revenues is driven by a jump in service taxes, with no observed discontinuity in property tax revenue. Following Cabral and Hoxby (2012), we argue that the decision to increase revenue through service taxes as opposed to property taxes is driven by salience. As service taxes are fragmented and collected as a small fraction of transactions at point-of-sale, they are less obvious to citizens and thus easier to collect.

To further identify the increased legislature and representation as the underlying mechanism driving these results, we run multiple placebo tests using various other population-based thresholds that could plausibly affect a municipality's public finances: federal transfers and local politician salaries. Employing a similar RDD strategy at these thresholds, we do not find any significant jump in taxes caused by either an increase in federal transfers or on mayoral salaries - suggesting that the makeup of the municipal council is driving the tax decision-making process.

Our results further suggest that higher legislature size and political representation increases municipal expenditures on "pro-social" public goods (health, education, and housing), consistent with the idea that improved representation enfranchises a larger share of the

population, whose demands and needs are met by their elected representatives.

Our paper contributes to two strands of the literature: the relevance of legislature size and the salience of taxes. The former is an old theme of political thought.[1] In a seminal work, Weingast, Shepsle and Johnsen (1981) construct a theoretical model showing that legislature size impacts government expenditure. They predicted that a larger number of legislators would be related to higher government expenditures as the former would have an incentive to increase government spending to please their core supporters at the expense of the general community. An empirical model testing this prediction was conducted by Pettersson-Lidbom (2012). The author uses data at the local level in Finland and Sweden and proposes an RD model taking advantage of a rule that assigned the number of members in the local council based on the population size. He finds that the local government size is negatively related to government expenditure, which is the opposite prediction of Weingast, Shepsle and Johnsen (1981). In our paper, we also use an RD model exploring a similar rule that determines the size of local council based on population. With a larger sample size and analyzing a different context, we show that municipal council size is positively related to tax revenue and consequently, higher government expenditures. The insights provided by our paper can therefore help one to understand the counter-intuitive result found in Pettersson-Lidbom (2012) and re-conciliate the theoretical prediction of Weingast, Shepsle and Johnsen (1981) with empirical evidence.[2]

This paper also relates to the literature on the importance of tax salience on the ability of (local) governments to raise tax revenue. John Stuart Mill, in 1848, already pointed out that the ability of governments to collect taxes depends on the underlying visibility in which these taxes are collected. The less visible indirect taxes, including services tax, might cause a "fiscal illusion", in which individuals underestimate the total tax burden as opposed to direct taxes, such as property taxes, a much more salient type of tax. Chetty, Looney and Kroft (2009) provides experimental evidence of heterogeneous behavioral changes in demand for different goods according to the salience of the tax in price tags across randomly selected grocery stores. Finkelstein (2009) provides empirical support to the idea that the salience of taxes are important to the determination of tax collection by comparing traditional cash-based toll rates with a less salient electronic toll collection (ETC), and finds that the introduction of ETCs has led to higher toll rates. On the political side of the issue, Sausgruber and Tyran

[1] As early as 1788, James Madison argued in *The Federalist Papers* No. 55 that the optimal number of members in the House of Representatives should not be fixed, but limited: the lower house should increase its size as more states were added to the union, however, it should be kept to a certain limit to enable coordination and avoid intemperance of discussion.

[2] The paper most closely related to this one isBritto and Fiorin (2019). In that paper, they examine the same discontinuity in local legislature size as we do, however their analysis focus on how council size affects corruption.

(2005) provides laboratory experimental evidence of misperception among subjects, whose vast majority vote for a redistributive tax policy framed as an indirect taxation scheme, which are not in their self-interest. Relatedly, Cabral and Hoxby (2012) exploit geographical differences in salience of property taxes through incidence of tax escrows in the US and find that less salience leads to higher property tax rates. In the case of Brazil, property taxes are perceived as a political burden to municipal legislators aiming to increase tax revenue, as opposed to the less salient services tax (Ter-Minassian, 2012).

The rest of the paper is organized as follows. Section I provides background on the election to and fiscal responsibilities of municipal councils in Brazil. Section II describes the data and section III discusses the empirical strategy. Estimation results, robustness checks, and extensions of the main empirical findings are presented in section IV, and section V concludes.

I Background

A Municipal Councils

The Brazilian political system is a federation with 26 independent states and the Federal District. Each state is composed of smaller administrative units, municipalities (*municípios*), with a local government comprised of an elected mayor (executive branch), municipal council (legislative branch), municipal court (judicial branch).

Municipalities enjoy some degree of political autonomy, guaranteed by the constitution. Specifically, municipalities enact their own laws (*lei orgânica*) and every fiscal year, the mayors elaborate a budgetary proposal detailing the public expenditure and investments for the subsequent year, based on the expected tax revenue. Members of the municipal council are responsible for evaluating, and voting the proposal, which after its approval, becomes the Budgetary Law (*Lei Orçamentária*) dictating the destination of the tax revenue collected at the municipal level.

The Law of Fiscal Responsibility (*Lei de Responsabilidade Fiscal*), enacted in 2000, imposes further restrictions to the municipal administration (on either executive, legislative and judiciary branches), limiting discretionary expenditures, including an upper limit of 60% of the total net revenue spent on personnel and total transparency on the destination of the resources.

Municipalities also collect taxes, which comprise services tax and property taxes. Service taxes (ISS) are levied on every transaction that involves the exchange of services within the municipality boundaries. Municipal counselors have the autonomy to legislate the tax

rate for any type of local service, as long as they respect the constitutional limits of 2% (minimum) and 5% (maximum). There are also two types of property taxes levied at the municipal level: property ownership tax (IPTU) and tax over real estate transactions (ITBI). Municipalities are only limited to set the maximum tax rates for property taxes.[3]

Municipalities have also the autonomy to the elect their political representatives. Municipal councils (*vereadores*) are directly elected by the population for a four-year term. However, unlike the mayor - who is elected by a simple majority rule – municipal councils are elected based on an open list proportional representation system, in which parties' share of seats is proportional to the quantity of votes cast to their candidates. Municipal councils, on average, have a wage that is 2.6 times the average in their municipalities (Colonnelli, Teso and Prem, 2017). Nonetheless, most of them have an outside job as they are only required to be in the council on average four days per month (Ferraz and Finan, 2011). Their main duty is to approve local legislation, such as tax adjustments and the municipal budget. They are also involved in the submission of bills and request for public works and monitoring the executive for its use of public resources.

B 2004 Municipal Council Reform

With the objective of reducing municipal expenditures on administration and personnel, in 2004, the Brazilian Supreme Electoral Court expedited a resolution assigning the number of seats to each municipality according to a strict rule based on population size: each municipality receives a minimum of 9 seats and additional seats according to increments of 47,619 inhabitants.

Specifically, municipalities with 47,619 inhabitants or fewer are assigned 9 seats, municipalities with population between 47,619 and 95,238 (47,619 + 47,619) inhabitants receive 10 seats, municipalities with population between 95,239 and 142,857 inhabitants receive 11 seats, and so on. Table 1 describes the assigned number of seats according to the established population thresholds for all population groups.

In practice, around 90% of the municipalities were assigned the minimum number of seats (9), compared to 56% before the reform. Moreover, 2,409 municipalities had the number of seats reduced, 3,120 municipalities did not experience any changes, and the number of seats increased in only 19 out of the 5,560 municipalities.

[3]The Brazilian law number 10,257 of 2001 limits the IPTU rate to be at most 15%. This value is much higher than the 1 to 1.5% rate that is usually charged across municipalities in Brazil and, therefore, is likely not biding.

II Data

We collect data, at the municipality level, on socio-economic variables (e.g., GDP and population); number of seats in legislative council; public spending; and electoral outcomes. Date on socioeconomic variables comes from the *Instituto Brasileiro de Geografia e Estatistica - IBGE* (Brazilian Institute of Geography and Statistics). Data on the number of seats in congress as well as electoral outcomes (e.g., votes for each municipal council candidate) were collected at the *Tribunal Superior Eleitoral - TSE* (Superior Electoral Court). Finally, public expenditures data comes from the *Secretaria do Tesouro Nacional - STN* (Brazilian National Treasury).

From IBGE, we collected yearly data from 2000 to 2010 on income and population. We are particularly interested in the 2003 Brazilian population, as the TSE decided to use this as the reference year to create population thresholds to assign the number of seats in municipal council that each municipality should have. This rule was based on the number of inhabitants each municipality had in 2003 to avoid any possibility of manipulating local population to obtain more (or less) seats in legislative council.

From TSE, we collect the number of seats in council to test whether the law was properly followed and we also collect electoral outcomes of municipal council elections to examine whether the number of effective parties changed in response to the new regulation.[4]

Finally, from STN we collected data on local taxation and expenditure in the areas of education, health and public employment.

III Empirical Strategy

To estimate the impact of council size on political diversity and taxation, we exploit the 2004 Supreme Electoral Court resolution that regulated the number of seats by implementing a strict rule based on the size of the population on each municipality. This setting provides a unique opportunity to implement a Sharp Regression Discontinuity Design (RDD), with council size as the treatment variable and population as the running variable. Focusing our attention to municipalities around the population cutoffs arguably produces as good as random assignment of council sizes. The underlying identifying assumption of our strategy, thus, is that municipalities with similar population levels around the cutoffs would have collected similar tax revenues in the absence of the 2004 resolution. The estimating equation can be expressed by

[4]See Laakso and Taagepera (1979) for a formal definition of effective number of parties.

$$Log(y_m) = \sum_{k=0}^{p} \lambda_k r_m^k + D_m \sum_{k=0}^{p} \gamma_k r_m^k + \boldsymbol{\theta}_m + \epsilon_m, \tag{1}$$

where y_m is the average tax revenue over the period between 2005–2008, which encompasses the complete mayoral and municipal council term for each municipality m; $D_m = \mathbb{I}(r_m \geq C_m)$ is the treatment variable indicating whether municipality m falls on the right side of a given population cutoff C_m, which we define it to be the closest population cutoff to municipality m within the thirteen cutoffs described in Table 1. The running variable is constructed by pooling the population cut-offs, normalizing them to zero, and defining $\tilde{r_m} = r_m - C_m$, such that $\tilde{r_m} > 0$ indicates treatment assignment, whereas non-treated municipalities fall on the negative side of the normalized cut-off (Cattaneo et al., 2016).

One potential issue associated with this pooling and normalization process is that municipalities close to different population cutoffs might differ in a range of characteristics (Bertanha, 2020; Cattaneo et al., 2016; Eggers et al., 2018). Since our estimation method produces an average of Local Average Treatment Effects (LATE) across all population thresholds, pooling municipalities with no covariate adjustments might introduce bias to our results. We address this concern by controlling for several observed municipality characteristics ($\boldsymbol{\theta}$ in our model), including population size in 2001, the average of population size between 2005 and 2008; number of council seats in 2001; and average GDP per capita between 2005 and 2008. We control for population numbers to capture the assignment to treatment rule, whereas council size in 2001 might be an important factor determining changes in the number of seats in 2004 (Mignozzetti and Cepaluni, 2019). Finally, GDP per capita might explain differences in the efficiency of the public sector, which might contribute to the ability of municipalities in collecting taxes. Besides improving consistency of estimates across multiple cutoffs, adding controls to our RD framework improves efficiency of our estimates Calonico et al. (2019).

We restrict our sample to observations within a bandwidth of size h on both sides around the normalized cutoff, such that $\tilde{r}_m \in [-h, h]$, to ensure that our results are not driven by unobservable factors associated with population size. We select the optimal bandwidth h based on Calonico, Cattaneo and Titiunik (2014), which proposes a robust data-driven method to account for biases arising from the choice of large bandwidths.[5] Finally, r is a smooth function of population size, which locally approximates its distribution around the cutoff. In our baseline specification, we employ a spline polynomial approximation of order up to p=2, given that higher order polynomials might introduce bias to our estimation

[5]The trade-off of choosing a smaller bandwidth is loss of precision due to a smaller sample (Calonico, Cattaneo and Titiunik, 2014)

(Gelman and Imbens, 2019).[6]

There are three main potential threats to the validity of our estimation strategy. First, our strategy relies on the assumption that the only difference between municipalities on either side of the cutoff should be the assignment to treatment (in our case, an additional seat in the municipal council). To address the plausibility of the assumption, we run our specification on a vast range of socioeconomic, municipal-level indicators to test whether there are any differences in these characteristics associated with the treatment assignment. Table 3 displays the results of these checks across several municipal characteristics, including public finance variables, electoral outcomes, and socioeconomic and demographic factors. As one can notice, we do not observe any statistically or economically significant differences on these characteristics across municipalities around the cutoff.

Second, if the population thresholds determining council size is also used for the implementation of other policies, the estimated coefficient will be biased. For example, federal transfers to municipalities (*Fundo de Participação dos Municípios – FPM*) are partly determined by municipal population size. Importantly, 3.6% of the federal transfers is allocated to municipalities with population size exceeding 142,633 inhabitants, which is close to one of the cutoffs in our specification. Therefore, our estimated coefficients might be partially capturing this effect. We address this concern by estimating a placebo treatment using the tax revenue collected in the previous term, between 2001 and 2004, when the 2004 electoral resolution was not in effect. If the population cutoffs applied in our model affect tax revenue through a channel different from council size, the placebo treatment should pick up this effect. We additionally test for different population thresholds to further check whether our results can be attributed to other population-based laws and policies. Specifically, we use population cutoffs that determine federal transfers as well as population thresholds that establish salary caps for legislators. Table 6 documents the results of the placebo treatments, using other policies' cutoffs (columns 1 through 6), as well as the falsification test applying our method to the 2000 elections term (columns 7 through 9). Reassuringly, we do not find evidence of statistically significant effects for any of the placebo treatments or falsification test described above.[7]

The third threat to the validity of our identification is the possibility of manipulative sorting by government officials on either side of the threshold. Our results might be biased if the likelihood to successfully manipulating official population numbers is correlated with

[6]We test for an ancillary of different specifications in Appendix A.II. Specifically, in figure A.2 we re-estimate our model to each year between 2005 and 2008 separately, and figure A.3 displays the results for different choices of Kernel densities. Our results are robust to these different specifications.

[7]See section IV C for a detailed discussion of these results.

(unobserved) municipal characteristics.[8] This concern is mitigated due to the fact that the 2004 resolution was based on 2003 population estimates, which was already published prior to the decision of the Supreme Electoral Court to establish the population-base rule of council size. Nevertheless, we formally address this concern by performing a density test to check for discontinuities in the population distribution around the cutoff, as suggested by McCrary (2008).

IV Results

A Electoral Diversity

The first set of results refers to the impact of legislature size on electoral diversity. According to the Duverger's law, single member district plurality elections favors two-party competition, i.e., if there is only one seat being disputed, then only two parties will be able to effectively compete in that election (Duverger, 1954). This can be explained by the psychological effects that the incentives provided in this voting system have on voters (Benoit, 2006). When voters realize that a third or minor party that they like most have no chance of winning, then they strategically vote for the party that they least "dislike" within the top 2 options.[9] Duverger (1954) has also proposed that multi-member proportional representation system, as the one used in Brazilian local legislation elections, favor multipartyism. In a laboratory experiment, Hix, Hortala-Vallve and Riambau-Armet (2017) show evidence of this prediction as they find that voting for the preferred party increases with district magnitude, i.e., number of seats.

We take advantage of the quasi-random increase in local council magnitude and examine whether Hix, Hortala-Vallve and Riambau-Armet (2017) findings can be extended beyond laboratory experiment and also holds in actual elections.[10] In Table 4 we use the effective number of parties as the dependent variable of our RDD estimation proposed in equation 1.[11] We find that one more seat in the legislature is translated in translated in 1.3 more effective

[8] Litschig (2012) finds evidence of politically motivated manipulation of official population numbers in the year 1991 in Brazil, suggesting that some municipalities enjoyed a larger share of the FPM on that particular year.

[9] Fujiwara (2011) shows empirical evidence of this prediction in Brazil. By analyzing a 200,000 eligible voters cutoff where, municipalities in the left-hand side elect their respective mayors using single-ballot, while the remaining municipalities use runoff plurality system, he shows that the former system makes voters disproportionately concentrate their votes within the top 2 candidates.

[10] For a study on the effect of district magnitude on the number of effective candidates, refer to Reed (1990) study of Japan showing that when there are n seats in the district, there are $n+1$ effective candidates.

[11] We follow Laakso and Taagepera (1979) and define the effective number of parties to be the inverse of the sum of the square of each party's proportion of all votes.

parties. Therefore, the range of ideological positions in local councils increased theoretically enfranchising voters' preferences that were not being previously represented in the legislative system.

B Tax Levels

Next, we turn to the question of whether the increased political representation from an additional seat has any effect on municipal tax revenues. As detailed in section III above, we define the main dependent variable of our analysis as the log of the yearly average tax revenue collected by municipalities during the electoral term 2005-2008. Figures 2, 3, and 4 show how municipal tax revenues change according to population size. Figure 2 displays the RD results for the total tax revenue collected in the corresponding period, indicating a clear discontinuity in the total tax revenue by municipalities above the normalized cutoff. The figure suggests that an additional seat increases total tax collection. This effect seems to be driven by an increase in services tax, whereas property taxes do not seem to be significantly different across municipalities around the cutoff, as documented in 3 and 4, respectively.[12]

Table 5 documents the regression estimates of the effect of municipal council size on tax revenue, illustrated in figures 2, 3, and 4. The RD results are constructed by local polynomial estimators of orders 1 and 2 with uniform kernel distribution. Columns (1) and (2) provides the results for total tax revenue, whereas columns (3) and (4) restricts the analysis to services tax and columns (5) and (6) to property tax. We find that on average, one additional seat increases total tax revenue by a magnitude between 50-54 percent (depending on the specification choice). This increase indeed seems to be capturing a higher incidence of services taxes as opposed to property tax. Our findings suggest an estimated increase of 61-66 percent in services tax, whereas point estimates for property tax is both not statistically and economically significant.

How large is the magnitude of this estimated increase in tax revenue? Although a 50 percent increase in tax revenue might seem an implausibly large number, the average share of service tax to total revenue is approximately 4.1 percent, whereas the average share of property tax is around 2.65 percent in our sample. The service tax revenue increase, thus, represents about 2 percent increase in total municipal revenue, on average. Consider a given municipality with the service tax revenue equal to the median value in our sample (R$ 1,314,093); the average total revenue of that municipality is R$ 60,098,110. The share of service tax revenue for this median municipality is, thus, around 2.19 percent. The estimated magnitude of the impact of an additional seat at the municipal council on service tax revenue

[12]Services and property taxes are the sources of revenue in which municipalities have discretionary power over the determination of their tax rate. See section I for further information.

is between R$ 805,539.00 to R$ 864,673.20 for this median municipality.

Our finding that an additional seat at the municipal council increases tax revenue mainly through service taxes and not property taxes is in line with the literature on the political economy of tax salience bias and welfare, in which consumers' response on tax incidence depends on the salience of the tax. Individuals are well aware of relevant tax information when it is drawn to their attention but, in general, fail to internalize it in making consumption decisions when that information is not explicitly visible, underreacting to less salient tax chages (Blumkin, Ruffle and Ganun, 2012; Chetty, Looney and Kroft, 2009; Finkelstein, 2009).

Although property taxes are economically more efficient to raise revenue than other types of taxes, given its immovable nature and, consequently, difficulty of evasion, they are relatively more salient than indirect taxes, such as services taxes (Slack and Bird, 2014).[13] We therefore interpret our results as an indication that, given the relatively higher political burden of an increase in property tax, legislators have an incentive to raise tax revenue through the less transparent services tax.[14]

C Robustness of Tax Results

To confirm that our results are not picking up effects of other cutoffs, we also look at run a model using variables based on the cutoffs for FPM transfer thresholds and legislator salary cap increase thresholds. These thresholds are documented in Rocha (2019) and Corbi, Papaioannou and Surico (2019) (FPM) and in Litschig and Morrison (2013) (legislative salaries). Note that our tests include additional thresholds not studied by Corbi, Papaioannou and Surico (2019). FPM transfers are a significant source of federal support for municipalities distributed within states proportional to a population-based coefficient, while the legislative salary caps dictate the maximum allowable wages for legislators; both of these policies are based on discontinuous functions of population which use different thresholds than the legislature size function. One might be concerned that municipalities just below (above) a threshold for change in the federal transfer coefficient could be more strained (free) in their financial resources, for which municipalities might compensate by taxing more (or due to which municipalities could be able to tax less). Likewise, legislators' ability to increase

[13] In Brazil, property tax payments are usually billed by the local authorities in the month of January to the property owner's address, as opposed to the indirect services tax, which are not explicitly displayed in the prices paid by the final consumer. Another important feature that distinguishes property tax from services tax is the fact that the former is levied as a single large tax relative to a given fiscal year, whereas the latter is collected in small amounts over the same fiscal year, which makes it difficult for consumers to fully account for the exact amount of yearly services tax paid.

[14] Bracco, Porcelli and Redoano (2019) finds a similar result for a relatively higher tax burden on less salient taxes in Italian municipalities when electoral competition is higher.

their salaries above thresholds on the salary cap could necessitate greater tax revenues to cover these increased expenses. Other authors have found effects on political competition and candidate quality from legislative salary thresholds (Ferraz and Finan (2011), Cunha and Manoel (2019)), and impacts on public goods provision from federal transfers (e.g., Litschig and Morrison (2013)). However, as seen in Table 6 our log tax variables do not register significant discontinuity at either of these thresholds.

We use the legislative term prior implementation of the legislative seat cutoff schedule (2001-4) as an additional placebo. This confirms that our strategy is well-specified and guards against the possibility that the 2003 population thresholds happened to align with preexisting discontinuities in taxes due to unobserved factors. In this specification, we use the log of average 2001-4 term total, service, and property taxes as outcome variables. The lack of significant results supports the clarity of our identification of legislative seat thresholds' effects in 2005-8.

D Public Expenditures

We start this subsection examining municipal expenditures to understand how the increase in tax revenue reported earlier is being used. We first study local public finances focusing on administrative costs as the increase in taxation could be motivated by individualistic purposes such as increasing the wages of municipal councils. However, our results in Table 7 show that legislative and administrative costs are not consistently changed suggesting that the extra revenue was not used to directly benefit municipal councils. The only significant result we find is an increase of close to 30 percent in public employment, which motivates us to examine whether local public services provision, such as education are affected. Examining the two largest social expenditures that municipalities are responsible to provide (i.e., health and education), we find a substantial increase of 32 percent in public expenditure in education and no change in health expenditures. Finally, when we examine urban development expenditure, which is associated with infrastructure projects that, according to Brollo and Nannicini (2012), are highly visible to voters, we find an increase of 42 percent in public expenditure in this sector.

Our results showing a considerable increase in tax revenues and expenditure in the areas of education and public employment is similar to the one reported in Schneider, Athias and Bugarin (2019). The authors examine the introduction of electronic voting in Brazil and show that, by *de facto* enfranchising poor voters, the new technology caused an increase in municipal tax revenue and expenditures. This result is consistent with the theoretical model constructed by Meltzer and Richard (1981) showing that, as poorer voters cannot be

excluded from using public goods but pay a smaller share of the tax revenue used to provide these goods, they are more in favor of larger taxation and public spending than relatively richer voters.[15] Although the legislation analyzed in this paper do not directly enfranchise voters as electronic voting did, we believe that, by increasing the number of seats in local councils the legislation allows voters to be indirectly enfranchised, i.e., their preferences have a larger likelihood of being represented as, theoretically, more parties can now effectively participate in elections increasing the range of ideological options.

V Conclusion

This paper generates significant insight in the role that political representation plays on political diversity and subsequent local tax collection. Relying on discontinuous changes in the number of seats allocated to municipal councils in Brazil, we find that one additional seat leads to higher tax collection and this effect is observed mainly through increased service tax revenue. This paper further documents that this is followed by a significant raise in social expenditures, namely education and housing. Our findings are consistent with the idea that legislature size is an important factor in shaping political diversity through an increased number of political parties being elected, effectively enfranchising a larger share of the population as predicted by Duverger's Law.

Assuming that incumbent local politicians' objective is to remain in office, devising mechanisms and institutional apparatus to represent voters' preferences is a rational response to achieve this goal. As more voters are represented in local councils, and by consequence, different groups of the population are enfranchised, local political actors' best response is to deliver policies that speak to this additional electorate, given the threat of non-reelection on subsequent elections. Our findings, thus, corroborate this hypothesis, given the significant increase in social expenditure on education and housing. Although we do observe some increase in administration costs, we do not find any significant systematic change in other administration expenditures such as personnel and legislative costs, which suggests no evidence of increases in expenditures for personal gains or clientelistic purposes.

Our paper provides support to potential benefits of multi-member proportional representation system, discussed in the context of Brazilian municipal legislation elections, as opposed to other types of election designs such as single member district plurality system, in devising the necessary institutional background to foster political diversity at the local

[15]See Fujiwara (2015) for an analysis of the impact of electronic voting on state level expenditures and improvements in public goods provision and see Schneider, Athias and Bugarin (2020) for an empirical examination showing a mechanism to explain how congressmen responded to electronic voting, namely by sending larger intergovernmental transfers to municipalities using the new technology.

level. Our paper sheds light on the intrinsic relationship between political representation and local-level state capacity, whereby larger shares of enfranchised voters might set the stage to greater ability to collect taxes and improved local public goods provision. Public policies aiming to improve these dimensions of local public finance might benefit from political reforms targeting greater political diversity and voters' enfranchisement.

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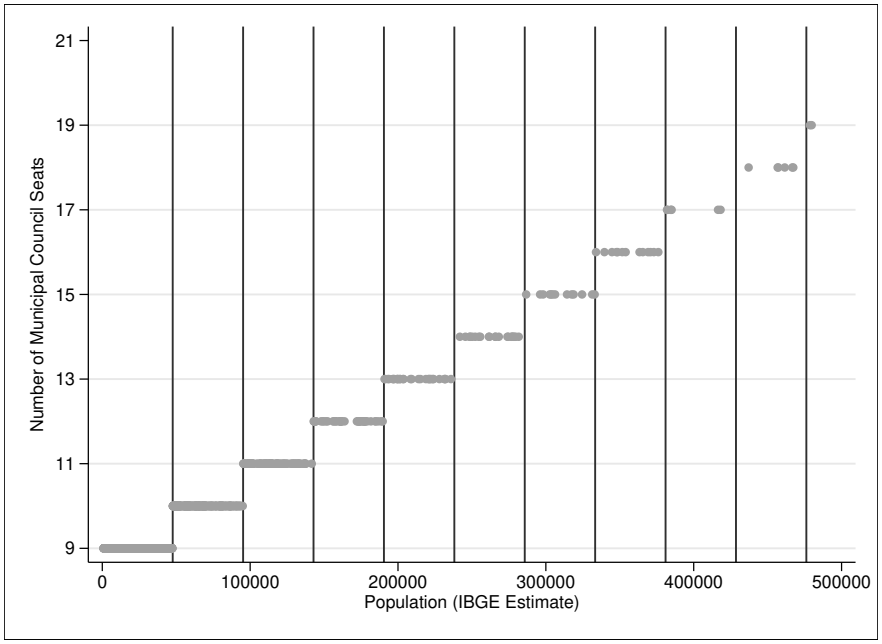
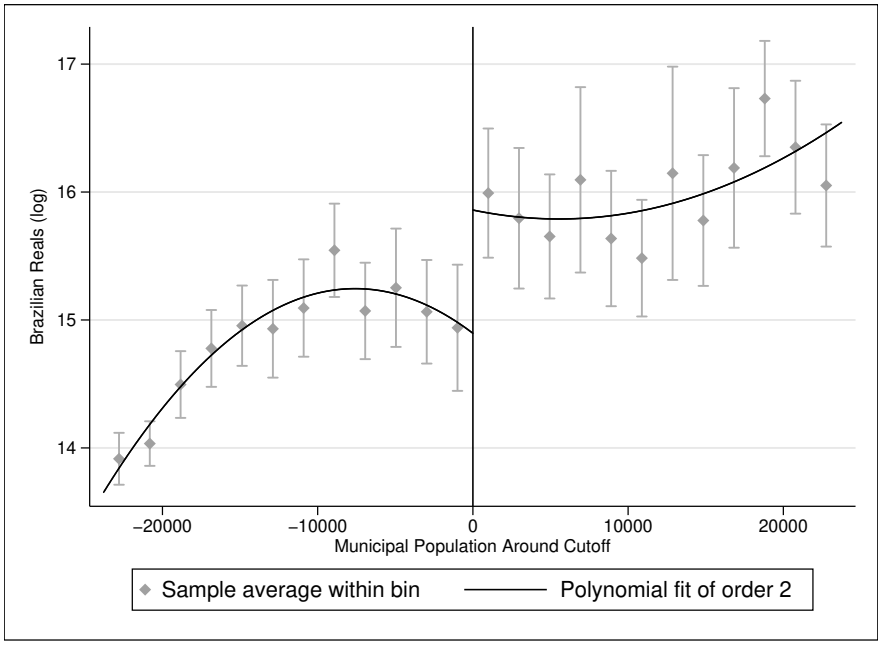
Figures

Figure 1: Municipal Council Sizes, 2005-2008



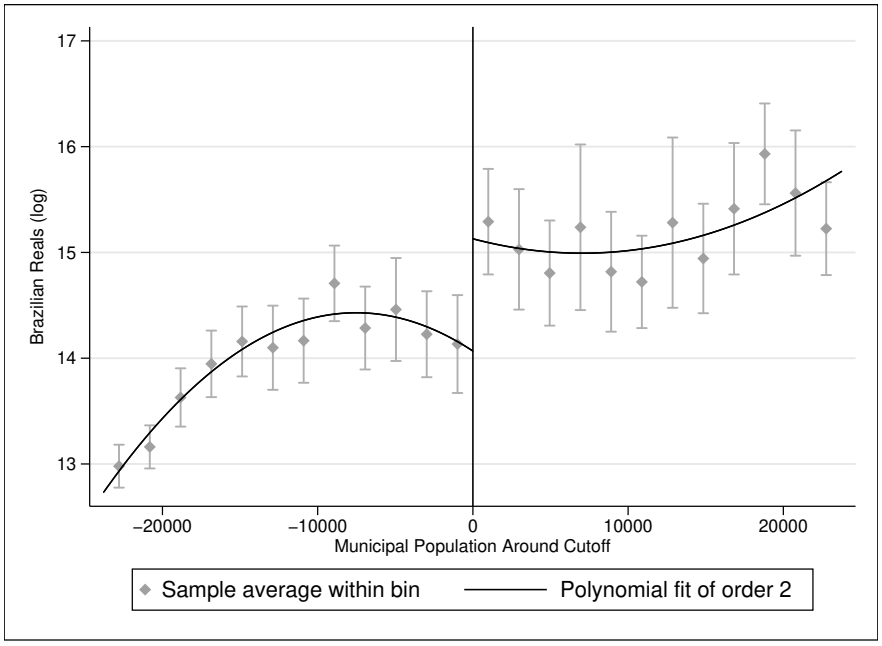
Notes: This figure shows the actual number of municipal council seats allocated to each municipality according to population size on the 2005–2008 electoral cycle.

Figure 2: Total Tax Revenue



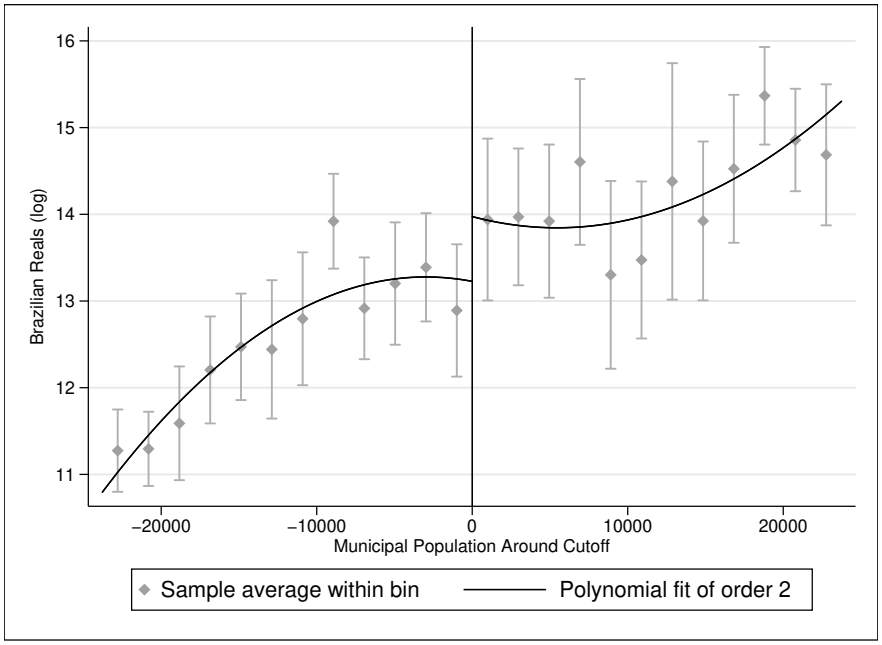
Notes: This figure shows the relationship between municipal council size and total tax revenue. RD estimates with locally smoothed quadratic polynomial with a uniform kernel are displayed. Each dot represents sample average tax revenue within each bin with 95% confidence intervals.

Figure 3: Service Tax Revenue



Notes: This figure shows the relationship between municipal council size and service tax revenue. RD estimates with locally smoothed quadratic polynomial with a uniform kernel are displayed. Each dot represents sample average service tax revenue within each bin with 95% confidence intervals.

Figure 4: Property Tax Revenue



Notes: This figure shows the relationship between municipal council size and property tax revenue. RD estimates with locally smoothed quadratic polynomial with a uniform kernel are displayed. Each dot represents sample average property tax revenue within each bin with 95% confidence intervals.

Tables

Table 1: Council Size Rule

Municipal Population	Number of Council Seats	Municipal Population	Number of Council Seats
[0 — 47,619]	9	[333,334 — 380,952]	16
[47,620 — 95,238]	10	[380,953 — 428,571]	17
[95,239 — 142,857]	11	[428,572 — 476,190]	18
[142,858 — 190,476]	12	[476,191 — 523,809]	19
[190,477 — 238,095]	13	[523,810 — 571,428]	20
[238,096 — 285,714]	14	[571,429 — 1,000,000]	21
[285,715 — 333,333]	15		

The table displays the population brackets and associated number of seats each municipality was allocated in the 2004 municipal legislature elections, after the 2004 TSE Resolution.

Table 2: Tax Revenue Summary Statistics

	Mean	Std Dev	Min	Max
Panel A: RDD Sample ($N = 1270$)				
Total Tax	17,069	61,082	126.8	966,980
Service Tax	8,371	30,589	40.90	396,516
Property Tax	5,400	21,003	0	356,857
Total Revenue *(,000 Reals)*	105,576	239,854	12,791	3,603,000
Revenue Share from Service Tax	0.0410	0.0367	0.002	0.320
Revenue Share from Property Tax	0.0265	0.0366	0	0.319
Panel B: CCT Bandwidth Sample ($N = 381$)				
Total Tax	22,792	68,207	306.2	767,338
Service Tax	11,483	35,914	116.4	395,450
Property Tax	7,009	23,255	0.340	236,693
Total Revenue *(,000 Reals)*	134,223	263,648	17,631	3,079,000
Revenue Share from Service Tax	0.0463	0.0419	0.004	0.320
Revenue Share from Property Tax	0.0295	0.0372	0	0.319

The table displays summary statistics of tax revenues for the municipalities in our sample. Panel A describes summary measures for the RDD sample, whereas Panel B restricts the sample to the municipalities within the bandwidth proposed by Calonico, Cattaneo and Titiunik (2014).

Table 3: Balance of Predetermined Covariates

	Linear ($p = 1$)		*Quadratic* ($p = 2$)	
	Coefficient	(SE)	Coefficient	(SE)
Panel A: Public Finance				
Municipal Income	-0.097	(0.73)	-0.034	(0.86)
Transfers from Federal Government	0.151	(1.10)	-0.422	(1.55)
Transfers from State Government	-0.829	(0.82)	-0.845	(1.11)
Panel B: Year 2000 Election				
Total Votes for Councilors	0.191	(0.18)	0.198	(0.21)
Number of Council Candidates	0.253	(0.19)	0.219	(0.24)
Voter Turnout	-0.011	(0.02)	-0.023	(0.02)
Council Seats (2001)	-0.405	(0.90)	-0.924	(1.08)
Panel C: Census Characteristics				
Geographic Area	0.129	(0.33)	0.108	(0.46)
Distance to State Capital	-0.234	(0.25)	0.051	(0.42)
Fraction of Households with...				
...running water	0.069	(0.06)	0.056	(0.08)
...electricity	0.039	(0.03)	0.036	(0.04)
...refuse collection	0.03	(0.04)	0.036	(0.05)
...car ownership	0.053	(0.05)	0.049	(0.05)
...land ownership	-0.005	(0.02)	0.015	(0.03)
Theil Inequality Index	-0.023	(0.03)	-0.049	(0.04)

The point estimates are constructed using local polynomial estimators with a uniform kernel. Significantly different than zero at 99 (***), 95 (**), 90 (*) percent confidence. Each column presents the RD specification restricting the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Taxes are measured $log(Reals)$. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.

Table 4: Effective Number of Parties

	Effective Number of Parties	
	(1)	(2)
LATE	1.008*	1.367*
	(0.524)	(0.701)
$N^-\|N^+$	917|356	917|356
$N_h^-\|N_h^+$	177|134	183|137
Bandwidth	7763	8198
Polynomial	1	2

The point estimates are constructed using local linear and local quadratic estimators with uniform kernels. Significantly different than zero at 99 (***), 95 (**), 90 (*) percent confidence. Each column presents the RD specification restricting the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Effective number of parties is measured as standard. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.

Table 5: RD Estimates

	Total Taxes		Service Taxes		Property Taxes	
	(1)	(2)	(3)	(4)	(5)	(6)
LATE	0.494***	0.537***	0.613***	0.658***	0.0727	0.221
	(0.125)	(0.144)	(0.145)	(0.162)	(0.265)	(0.356)
$N^-\|N^+$	912|356	912|356	912|356	912|356	912|356	912|356
$N_h^-\|N_h^+$	155|124	253|170	168|130	300|193	209|150	234|163
Bandwidth	6843	10081	7397	11460	8820	9566
Polynomial	1	2	1	2	1	2

The point estimates are constructed using a local polynomial estimator with a uniform kernel. Significantly different than zero at 99 (***), 95 (**), 90 (*) percent confidence. Each column presents the RD specification restricting the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Taxes are measured $log(Reals)$. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.

Table 6: Placebo Tests

	Placebo Test: Legislator Salary Threshold						Placebo Test: Federal Transfers Threshold					
	Total Taxes		Service Taxes		Property Taxes		Total Taxes		Service Taxes		Property Taxes	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
LATE	0.0930	0.0935	0.0467	0.0372	0.267	0.416						
	(0.111)	(0.156)	(0.141)	(0.192)	(0.261)	(0.294)						
$N^-\|N^+$	585|381	585|381	585|381	585|381	585|381	585|381						
$N_h^-\|N_h^+$	377|193	430|210	362|191	454|218	188|124	386|195						
Bandwidth	17045	18328	16607	18944	10277	17302						
Polynomial	1	2	1	2	1	2						
LATE							-0.311	-0.397	-0.581*	-0.630	-0.106	-0.396
							(0.281)	(0.387)	(0.349)	(0.466)	(0.225)	(0.289)
$N^-\|N^+$							955|143	955|143	955|143	955|143	955|143	955|143
$N_h^-\|N_h^+$							53|49	67|53	49|41	69|56	53|47	57|49
Bandwidth							1575	1852	1460	1946	1565	1584
Polynomial							1	2	1	2	1	2

The point estimates are constructed using a local quadratic estimator with a uniform kernel. Significantly different than zero at 99 (***), 95 (**), 90 (*) percent confidence. Each column presents the RD specification restricting the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Taxes are measured $log(Reals)$. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.

Table 7: Government Expenditure Changes

	Panel A: Administrative Expenditures							
	Administration		Legislative Costs		Total Labor		Security Forces	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LATE	0.287**	0.317*	1.131	1.184	1.980	1.989	0.556	0.895
	(0.144)	(0.170)	(0.911)	(1.072)	(1.218)	(1.550)	(0.978)	(1.420)
$N^-\|N^+$	912|356	912|356	912|356	912|356	912|356	912|356	912|356	912|356
$N_h^-\|N_h^+$	148|124	234|163	122|116	184|138	163|129	201|149	246|168	221|159
Bandwidth	6657	9578	6137	8249	7091	8601	9918	9200
Polynomial	1	2	1	2	1	2	1	2

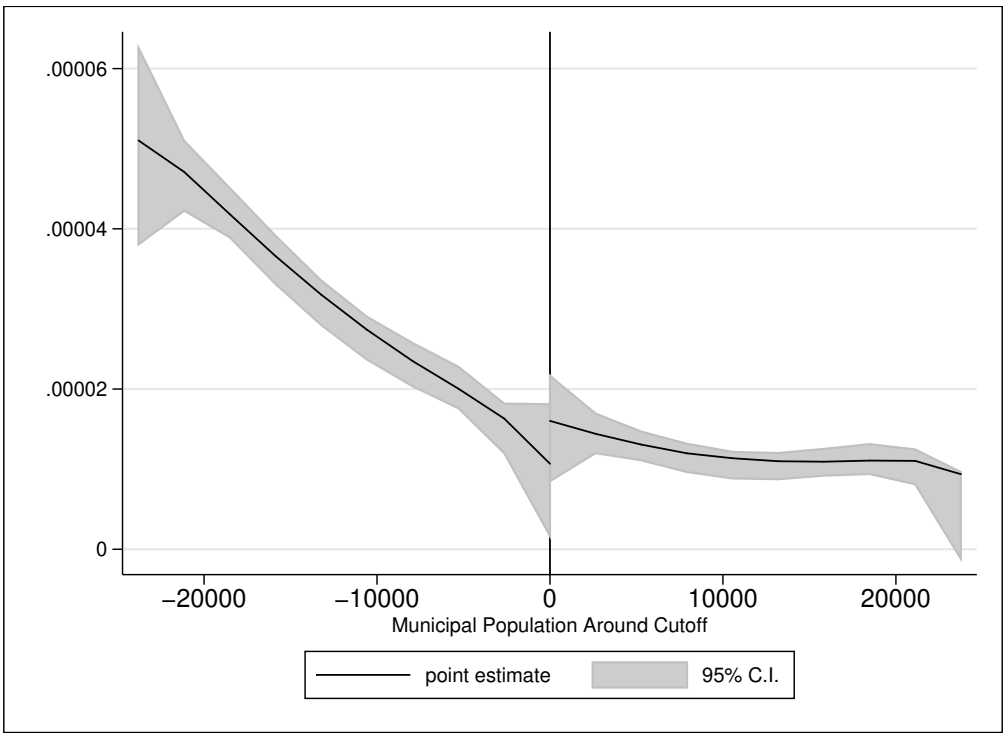
	Panel B: Social Expenditures							
	Education		Health		Housing		Social Assistance	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
LATE	0.250***	0.291**	0.133	0.162	0.545***	0.607***	0.319	0.513*
	(0.0934)	(0.116)	(0.107)	(0.125)	(0.160)	(0.213)	(0.228)	(0.310)
$N^-\|N^+$	912|356	912|356	912|356	912|356	912|356	912|356	912|356	912|356
$N_h^-\|N_h^+$	134|119	200|145	153|124	205|149	164|130	188|140	176|134	181|135
Bandwidth	6406	8520	6788	8713	7158	8339	7743	7934
Polynomial	1	2	1	2	1	2	1	2

The point estimates are constructed using local linear and local quadratic estimators with uniform kernels. Significantly different than zero at 99 (***), 95 (**), 90 (*) percent confidence. Each column presents the RD specification restricting the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Taxes are measured $log(Reals)$. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.

A Appendix

A.I McCrary Density Test

Figure A.1: Manipulation Test Plot



The figure shows the results of the McCrary (2008) manipulation test with p=2, q=3

Table A.1: Manipulation Test

	Bandwidth		Observations		t-Test	
	left	right	left	right	t-test	p-value
$T_1(h_1)$	3628.24	3891.11	63	75	0.039	0.968
$T_2(h_2)$	12314.57	12100.28	325	202	0.957	0.338
$T_3(h_3)$	8942.21	9817.64	215	167	-1.056	0.291
$T_4(h_4)$	24689.24	20695.38	920	323	1.384	0.166

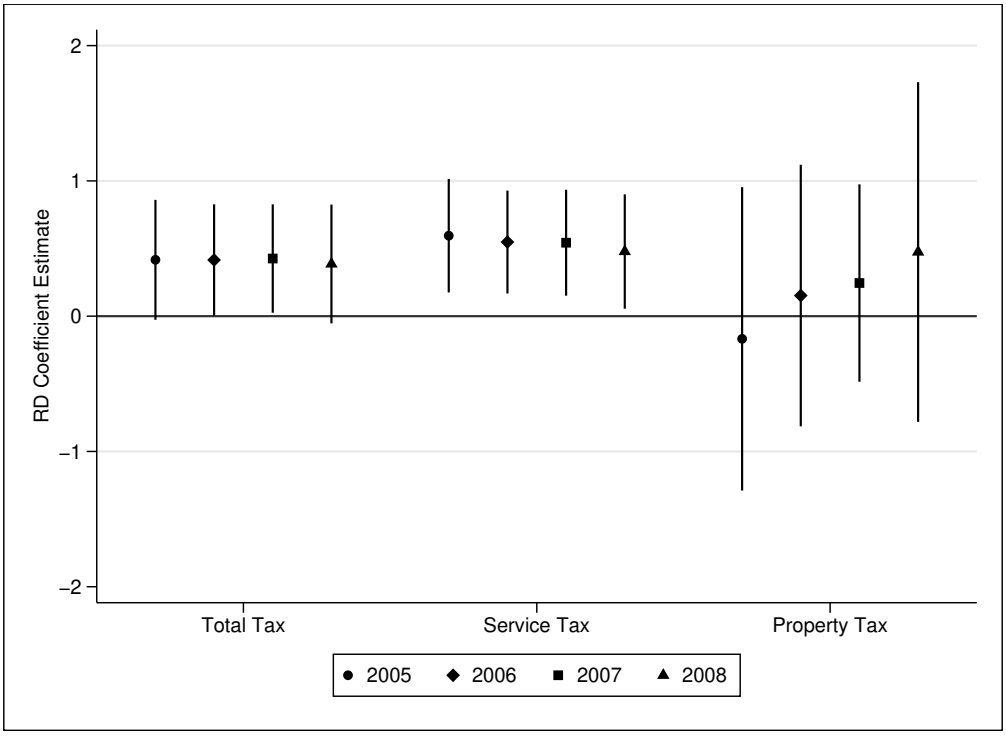
$T_p(h)$ is the manipulation test statistic of bandwidth h. p indicates the order of the local polynomial used to construct the density point estimator.

A.II Robustness to RD Specification

Yearly Coefficients

Figure A.2 displays the RD estimates for each year of the 2005–2008 electoral cycle. The estimates show a fairly similar effect of an additional seat on total taxes across the analyzed period. The estimates further corroborate the aforementioned results that the effects are mainly driven by service tax, with no consistent effects on property tax.

Figure A.2: Point Estimates for Each Year 2005 - 2008

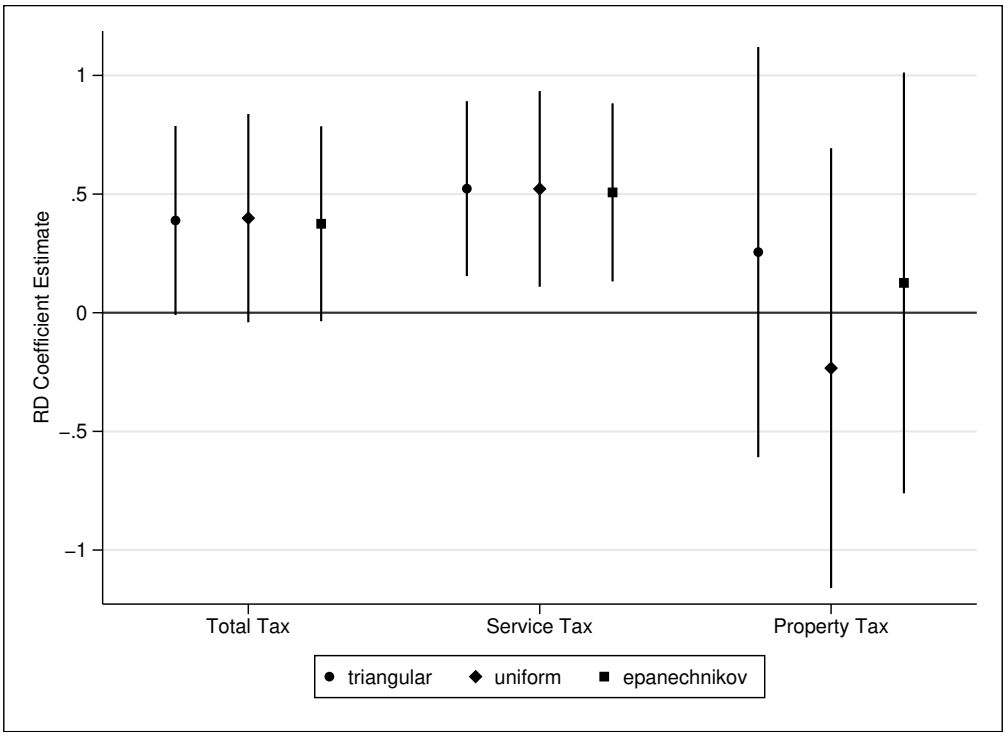


Notes: This figure shows RD estimates for each year of the electoral cycle (2005–2008). The point estimates along with 95 percent confidence intervals are constructed using local linear and local quadratic estimators with kernels specified by each symbol displayed in the legend. Each RD specification restricts the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Taxes are measured $log(Reals)$. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.

Choice of Kernel

Figure A.3 provides estimates of the RD model for different kernel densities. Specifically, we compare estimates of our main specification (uniform) for the effects of municipal council size on total tax revenue, service tax, and property tax with alternative specifications: triangular and Epanechnikov densities. The results are similar across kernel choices, alleviating concerns related to the specific choice of kernel.

Figure A.3: Point Estimates and Kernel Choice



Notes: This figure shows RD estimates for three different kernel choices: triangular, uniform, and epanechnikov. The point estimates along with 95 percent confidence intervals are constructed using local linear and local quadratic estimators with kernels specified by each symbol displayed in the legend. Each RD specification restricts the sample to bins of size h around the municipal population cut-off. Results shown using the optimal bandwidth h developed by Calonico, Cattaneo and Titiunik (2014). All specifications include additional controls for GDP per capita, population, pre-2004 council size, and average population from 2001-2004. Taxes are measured $log(Reals)$. $N^+ = \sum_1^n 1(c \leq X_i)$, $N^- = \sum_1^n 1(X_i \leq c)$, $N_h^+ = \sum_1^n 1(c \leq X_i \leq c+h)$, $N_h^- = \sum_1^n 1(c-h \leq X_i \leq c)$.